Separable Equations

In this section, we examine a subclass of linear and nonlinear first order equations.

Consider the first order equation $\frac{dy}{dx} = f(x, y)$

- We can rewrite this in the form

$$M(x, y) + N(x, y)\frac{dy}{dx} = 0$$

- In differential form, $M(x, y)dx + N(x, y)dy = 0$
- If M is a function of x only and N is a function of y only, then $M(x)dx + N(y)dy = 0$
- In this case, the equation is called **separable**.

Implicit Solution of Initial Value Problem

Consider the following initial value problem :

$$y' = \frac{2y\sin x}{1-2y^2}, \quad y(0)=1$$

- Separating variables and using calculus, we obtain

$$\frac{1-2y^2}{y}dy = 2\sin x dx$$

$$\int\left(\frac{1}{y}-2y\right)dy = 2\int \sin x dx$$

$$\ln|y| - y^2 = -2\cos x + C$$

- Using the initial condition, it follows that

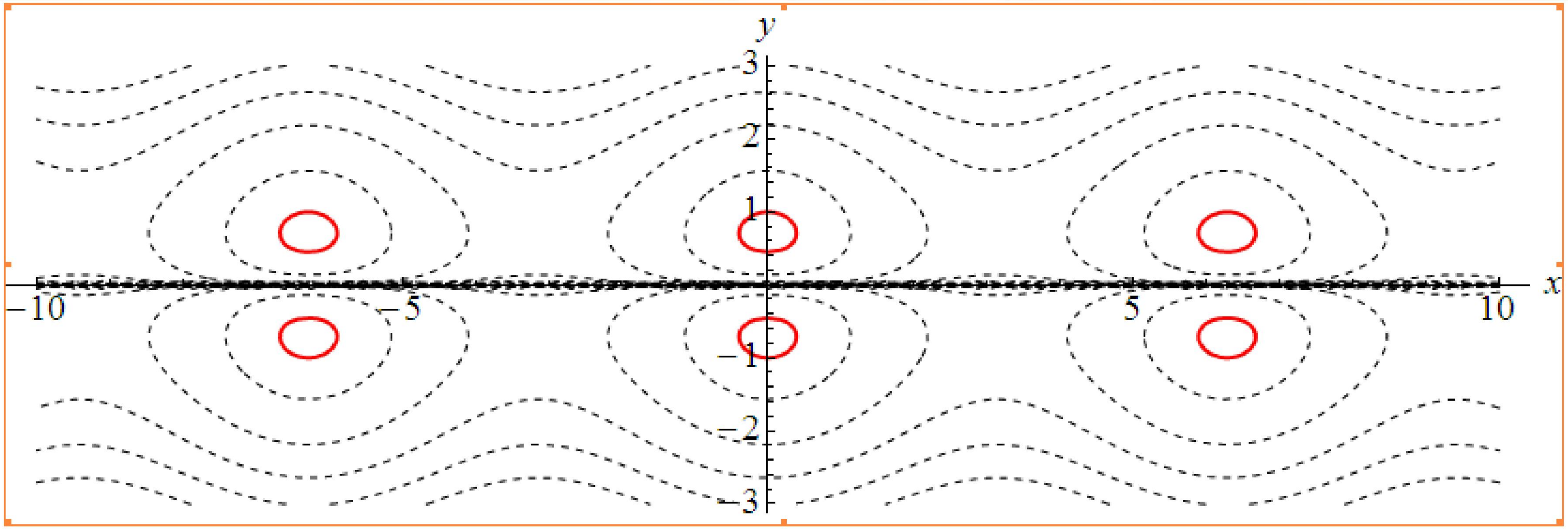
$$\ln y - y^2 = -2\cos x + 1$$

Graph of Solutions

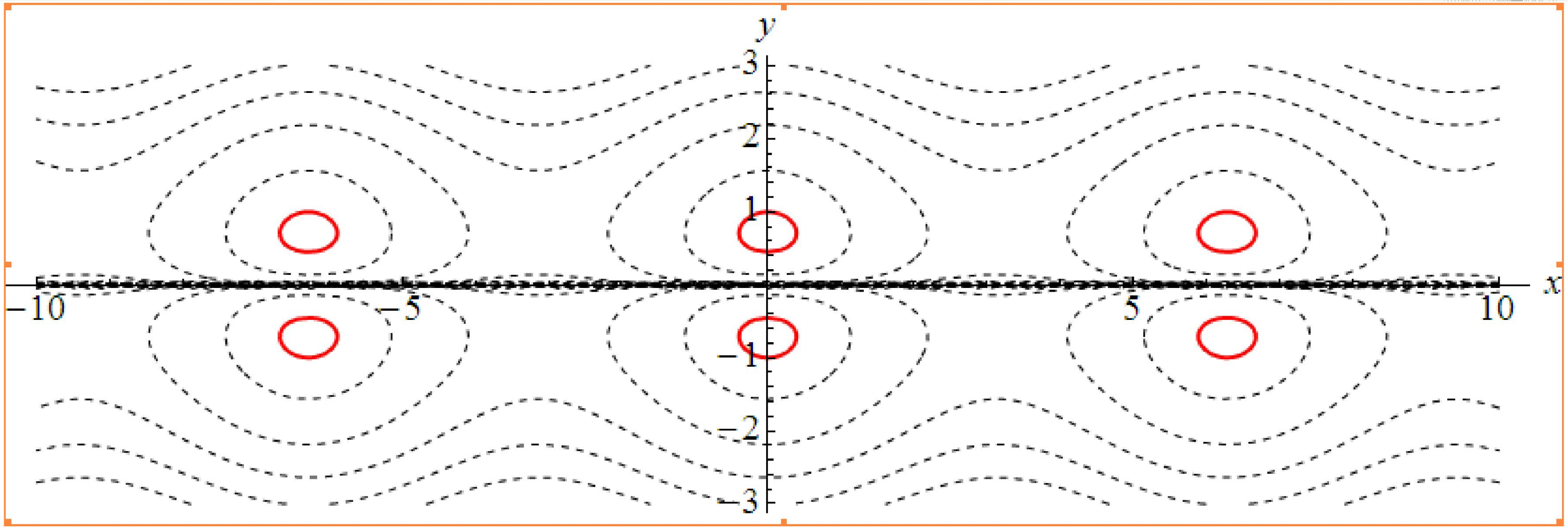
- Thus,

$$y' = \frac{2y\sin x}{1-2y^2}, \quad y(0)=1 \Rightarrow \ln y - y^2 = -2\cos x + 1$$

- The graph of this solution (red line) and several integral curves (black dashed line) for this differential equation, is given below.



Mathematica Code




```
A = ContourPlot[ Log[Abs[y]] - y*y + 2 Cos[x], {x, -10, 10}, {y, -3, 3},
      ContourShading -> None, ContourStyle -> {Dashed},
      AspectRatio -> Automatic]
B = ContourPlot[Log[Abs[y]] - y*y + 2 Cos[x] == 1, {x, -10, 10}, {y, -3, 3},
      ContourShading -> None, ContourStyle -> {Red, Thick},
       AspectRatio -> Automatic]
Show[A, B]
```