SEOUL
Intelligencer

International Congress of Mathematicians
August 13-21 | 2014 | Seoul

SEOUL ICM 2014

Springer

Sung Sa Hong
Sogang University

Young Wook Kim
Korea University

A Brief History of Korean Mathematics

1 History of Korea

Korea has a long history which begins with an ancient kingdom Gojoseon, literally an old Joseon, established in the 23rd century BCE. An extant record of Gojoseon is found in *Samguk Yusa* (ca. 13th century), a history of the Korean dynasties before the Goryeo dynasty, published by the Buddhist monk Il Yeon. Some records of Gojoseon are also seen in Chinese literature such as *Shanhai Jing* (ca. 4th century BCE), *Shiji* (109–91 BCE) of Sima Qian, and *Hanshu* (111 BCE). After the fall of Gojoseon in 108 BCE, there existed several tribal states in this territory which gradually was formed into four countries, Goguryeo, Baekje, Silla, and Gaya, but known as the Three Kingdoms of Korea, and which lasted 500–700 years. Between 660–676, Silla absorbed Baekje and Goguryeo in succession and established Unified Silla (668–935). In the remaining territory of Goguryeo, the wandering people from Goguryeo established a new country, Balhae (698–926). After the fall of Unified Silla came Goryeo (918–1392). The current name 'Korea' comes from the name 'Corea,' which was the transliteration of 'Goryeo' in Europe. In 1392, Goryeo was succeeded by the next dynasty, Joseon which lasted until 1910.

Still existing major historical records of Korea are *Samguk Sagi* (1145), a history of the Three Kingdoms; *Samguk Yusa*, *Goryeosa* (1451), a history of the Goryeo Dynasty; *The Annals of the Joseon Dynasty* (abbreviated as *The Annals*) or *Joseon Wangjo Sillok*; and the *Journal of the Royal Secretariat* or *Seungjeongwon Ilgi*.[1] *Samguk Sagi* and *Samguk Yusa* are based on old but now lost records.

2 Ancient and Medieval Mathematics of Korea

2.1 Three Kingdoms and Unified Silla

We have very limited information about the mathematics of the early ages of Korea. According to the remaining records, Goguryeo recorded its own history, *Yugi*, from the early stage which presumably recorded astronomical data. Baekje began recording its history, *Seogi*, in the mid-4th century, and Silla, *Guksa* in the mid-5th century, respectively. All of these texts are lost, but their records might have appeared in *Samguk Sagi*, as was mentioned above.

Silla established its educational system in 682, namely, the national school *Gukhak*, which included mathematical education among others. Silla absorbed the systems of Goguryeo and Baekje and modeled its system after that of the Tang dynasty of China. They taught the courses *Cheolgyeong*, *Samgae*, *Gujang* and *Yukjang*.[2]

2.2 Goryeo

Among the well-known cultural heritages of Goryeo are their celadon porcelains and the invention of metal type for printing. Goryeo was very close politically and culturally to the Song dynasty of China, which had the most flourishing civilization and also the most advanced mathematics in the history of China. Goryeo imported a large quantity of books from China and also printed many books. The record of the first use of metal type is the printing of *Sangjeong Gogeum Yemun*, or *Book on Old and Contemporary Rites* in 1234. The oldest extant printing that was set in metal type is *Jikji Simche Yojeol* or *Anthology of Great Buddhist*

[1] *The Annals* and the *Journal* are both registered in UNESCO's *Memory of the World Programme* (see http://sillok.history.go.kr).
[2] *Cheolgyeong* is from China and is called *Zhuijing* in Chinese pronunciation. *Samgae* literally means 'extraction methods of three kinds'. *Gujang* is the *Jiuzhang Suanshu*, and *Yukjang* means 'the six chapters'.
[3] UNESCO confirmed that *Jikji* used the world's oldest metal type and includes it in the *Memory of the World Programme.*

Priests' Zen Teaching of 1377, also known simply as *Jikji*.[3] This precedes Gutenberg type (1440) by more than 60 years.

In the mid-10th century, Goryeo introduced the system of national examinations for selecting government officials, including mathematical positions. The subjects for mathematical officials include *Gujang*, *Cheolsul*, *Samgae*, and *Saga*. The first three subjects are identical to those of Unified Silla. The national educational system of Goryeo began with *Gukjagam*, a national university, in 992. This system consisted of six schools, namely, *Gukjahak*, *Taehak*, *Samunhak*, *Yulhak*, *Seohak*, and *Sanhak* (mathematics). They had the positions of *Baksa*, literally a doctor, and *Jogyo*, an assistant, and the curriculum of the mathematics school (*Sanhak*) was for six years.

These systems were basically retained in the next dynasty, Joseon.

3 Mathematics of Joseon

Based on historical records of Joseon, also known as Chosun, we are reasonably certain that among the 13th–15th century Chinese mathematical books, the only ones imported to Joseon were *Yanghui Suanfa* (1274–1275) by Yang Hui, *Suanxue Qimeng* (1299) by Zhu Shijie (fl. end of the 13th century), and *Xiangming Suanfa* (1373) by An ZhiZhai. These formed the subjects of the examination for selecting mathematical officials in Joseon since King Sejong (1419–1450), the fourth king of the Joseon dynasty, and they were reprinted several times in the whole period of Joseon.

3.1 Science in Joseon

The first 100 years of Joseon showed a multitude of achievements in diverse areas of science. Among those remaining today are ones dealing with astronomical instruments and scientific documents related to calendars, but not the mathematical ones.

King Sejong encouraged his officials and scholars to study mathematics, astronomy, and medicine so that science advanced greatly under his reign. To name only a few achievements, *Hangul*, the Korean alphabet, was invented; and *Cheugugi*, the world-premier rain gauge, was invented, along with many astronomical instruments. King Sejong regarded the calendar as important for agriculture and developed the Korean calendar system, *Chiljeongsan* (1444), based on the Chinese one. In this process, Chinese mathematics was extensively studied and improved. It is recorded in *The Annals* that the experts of the time could use the methods of *Suanxue Qimeng* to solve polynomial equations of degrees up to 10 and that King Sejong himself studied the *Suanxue Qimeng*.

Moreover, the technology of metal type had improved greatly, resulting in very active publication of books in the printing office, *Gyoseogwan*. In military science, various types of weaponry were developed, including explosives.

Kim SiJin (1618–1667) republished *Suanxue Qimeng* in 1660, giving a great impetus for the restoration of devastated Joseon traditional mathematics that had resulted from the foreign invasions in 1592 and 1636.

In the middle of the 17th century a new calendar system, *Shixian Li* (1645), influenced by Western astronomy and mathematics, was introduced into Joseon. For that system, new knowledge from the West streamed into Joseon through China, notably: *Xiyang Xinfa Lishu* or *The Treatise on (Astronomy and) Calendrical Science According to the New Western Methods* (1645) and *Tianxue Chuhan* (1623) compiled by Li ZhiZao and including *Jihe Yuanbe* (1607), the translation of Euclid's *Elements* by Ricci and Xu Guangqi, and *Tongwen Suanzhi* (1613), during the 17th century. Also, *Shuli Jingyun* or *Collected Basic Principles of Mathematics* (1723), a part of *Luli Yuanyuan* or *Ocean of Calendrical and Musical Calculations*, was published under the command and supervision of the Emperor Kangxi and was introduced in the mid-18th century. This stimulated Korean scholars, mostly mathematicians of the noble class and those in the state observatory, Gwansanggam.

3.2 Mathematics of Early Joseon

We have almost no concrete records of mathematics of Joseon before the 17th century, but according to what is written in *The Annals* we can infer the circumstances. Joseon showed enormous advances under King Sejong's reign, as mentioned above, but these achievements had already been deprecated by the second half of the 15th century.

In an article in *The Annals* of the King Sejo era (1460), the historian who drafted it quotes important mathematical statements from *Suanxue Qimeng* and those for *Haidao Suanjing* in *Yanghui Suanfa*, and then gives the essential motives for the development of Joseon mathematics. He notes that, in spite of difficulty in importing books, thanks to King Sejong's effort they were able to establish their mathematics and astronomy. The article indicates that mathematics in the early Joseon dynasty was developed along with the study of astronomy.

4 Mathematics of Joseon After the 17th Century

4.1 Characteristics of Joseon Mathematics

Composition of Mathematicians. We do not know of any significant mathematical works in Joseon until the 17th century. In *Juhak Ibgyeokan*, we have a record of 1,626 mathematical officials in the department Hojo (Department of Home Affairs and Finance) who passed the examination from the end of the 15th century to 1888. According to the record, the candidates were restricted to members of the

Jungin[4] families and hence the jobs were passed down generation after generation in the same families. Since Hojo dealt with census, taxation, national accounting, and economy, mathematical officials in Hojo were not required to study pure mathematics.

The study of astronomy was carried out by officials in Gwansanggam who were appointed through a national examination. Although they were also from the Jungin families and studied mathematics as well, we find no evidence of communications between the two groups, one from Hojo and the other from Gwansanggam.

All the Joseon mathematics books quoted in this article, can be found in [4]. The general facts of the history of Chinese mathematics is well explained in [5].

Works of Jungin Mathematicians. Only three mathematicians from the Jungin families produced mathematical works, namely, *Muksajib Sanbeob* by Gyeong SeonJing (1616–?), *Guiljib* (1724) by Hong JeongHa (1684–?); and *Chageunbang Monggu* (1854), *Sansul Gwangyun* (1855) and *Iksan* (1868) by Lee SangHyeok (1810–?). Since they were all related by marriages, transcribed copies of their works should have been available among themselves.

Among these works, *Guiljib* was highly appreciated by Lee SangHyeok and Nam ByeongGil (1820–1869) more than one century later. But with this one exception, mathematics by Gyeong SeonJing and Hong JeongHa were completely ignored by mathematicians of the noble class.

Works of Yangban Mathematicians. Mathematics was also studied by those from the Yangban families, consisting of civil and military nobility. Among them, we list the mathematicians of the noble class who wrote significant mathematical works, as follows: Park Yul, Choe SeokJeong (1646–1715), Jo TaeGu (1660–1725), Nam ByeongChul (1817–1863), Nam ByeongGil (1820–1869), Jo HeeSoon (ca. mid-19th century), all of whom served as government officials, and Hwang YoonSeok (1729–1791), Hong DaeYong (1731–1791), and Hong GilJu (1786–1841), who were scholars devoted to writings on various subjects.

Collaboration of Nam and Lee. Nam ByeongGil and Lee SangHyeok served together in Gwansanggam. Nam noticed Lee's great talent, and despite their difference in social status they became mathematical collaborators, unique in Joseon, and produced many significant results.

Nam ByeongGil was fully equipped with most of the Chinese classics including *Jiuzhang Suanshu*, books published in the Song-Yuan and Qing dynasties, together with books influenced by Western mathematics. They pointed out a logical flaw in the method of solving equations in *Shuli Jingyun*.[5] They clearly understood the superiority of the theory of equations in the Song-Yuan era.

In the following section, we present interesting mathematical results related to mathematical structures that show the mathematical characteristics of Joseon mathematics [2].

4.2 Theory Related to Numbers

Choe SeokJeong. Choe SeokJeong served as the highest official Yeongeuijeong, the prime minister. Extending magic squares and the like in *Yanghui Suanfa*, he contributed remarkably to the study of magic squares and related topics, in particular obtaining an orthogonal Latin square of order 9 in his *Gusuryak* (ca. 1715–1718?). It was much earlier than Euler.

Figure 1. The orthogonal Latin square of order 9 as found in Choe SeokJeong's *Gusuryak*.

Hong JeongHa characterized least common multiples as follows:
For $a_1, a_2, \dots, a_n \in \mathbb{N}$, their least common multiple l is precisely the number $l = a_i c_i$ for some relatively prime c_i's $(i = 1, 2, \dots, n)$.

Also Hong GilJu (1786–1841) found formulae for representing n^k $(k = 2, \dots, 5)$ by triangular numbers and sum of squares which seem hard to find [2].

Lee SangHyeok. Lee SangHyeok studied the Commentary on Zhu Shijie's *Siyuan Yujian* (1303) by Luo ShiLin (1774–1853), which contains the theory of finite series including Zhaochashu (the method of finite differences), and then wrote a book *Toetaseol* (*Theory of Series*), the second book in his last work *Iksan*.

He also introduced the concept of Jeoljeok, i.e., a truncated sum $\sum_{i=m}^{n} a_i$ $(m > 1)$. To get truncated sums, he introduced Bunjeokbeob (Splitting Method; See Figure 2).

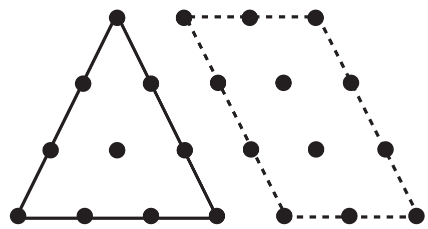

Figure 2. Splitting Method

[4] A class between the nobles and the common people.

[5] Every Joseon mathematician who studied *Shuli Jingyun* disregarded its poor method of solving equations.

Using these, he organized a theory of finite series according to mathematical structures and a primitive version of mathematical induction in *Toetaseol*.

4.3 Geometry and Verifications

Before the 18th century, Yang Hui's proof for *Haidao Suanjing* (in *Yanghui Suanfa*) was probably the only example of the verification of geometrical statements for Joseon mathematicians. Jo TaeGu (1660–1725) studied Euclid's *Elements*, *Jihe Yuanben* (1607), translated in China, and introduced similar right triangles and their properties in his book *Juseo Gwangyun*. Jo was the first Joseon mathematician who included explicit proofs for his statements. Nam ByungGil also used them in his *Cheuklyang Dohae* (1858) as well.

Lee SangHyeok dealt with general triangles that are similar or congruent and the geometry of circles in his *Sansul Gwangyun* (1855). His proofs in the book are based on logical reasoning which is exactly the same as those in demonstrative geometry books of the present day.

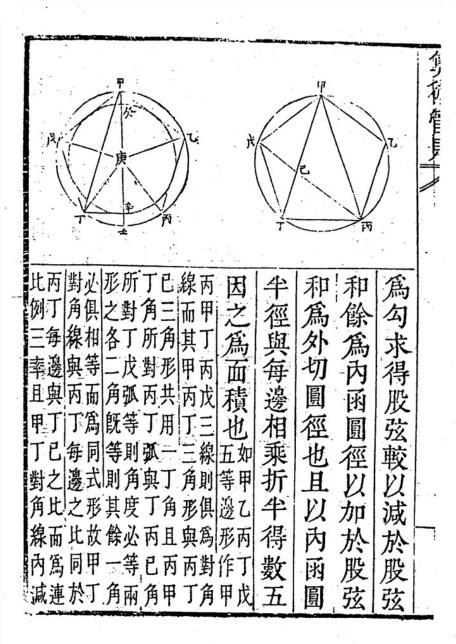

Figure 3. A geometric problem in *Sansul Gwangyun* (1855) by Lee SangHyeok. This problem is to find the diameters of the circumcircle and the incircle of a regular pentagon with geometrical demonstrations.

Hong DaeYong (1731–1791) was the first mathematician to introduce trigonometry in his *Juhae Suyong*, which did not get any attention for the lack of the trigonometric table.

4.4 Theory of Equations

The theory of equations is the most important subject in Joseon mathematics as in Chinese mathematics. The theory of equations of higher degrees is divided into two parts; one is construction of equations and the other is solving them. For the former, Chinese mathematics developed Tianyuanshu, which is a method to represent polynomials by the sequences of their coefficients and to manipulate their operations. For the latter, Shisuo Kaifangfa, a method of extracting roots using geometric binomial expansions was given in Juizhang Suanshu. This method was developed and improved to Zengcheng Kaifangfa in the 11th–13th centuries. Zengcheng Kaifangfa is a synthetic divisions method. This new method is similar to the Ruffini–Horner method of the 19th century, but it was developed from a totally different context.

Before the 19th century, *Sanhak Wonbon* (1700) by Park Yul (1621–1668) is a unique printed book with the publication year in Joseon. Moreover, it is the first book in Asia dealing with Tianyuanshu after Zhu Shijie's *Siyuan Yujian* (1303). Park had a modern structural viewpoint so that he used Tianyuanshu for constructing all equations, even including linear equations in it [3].

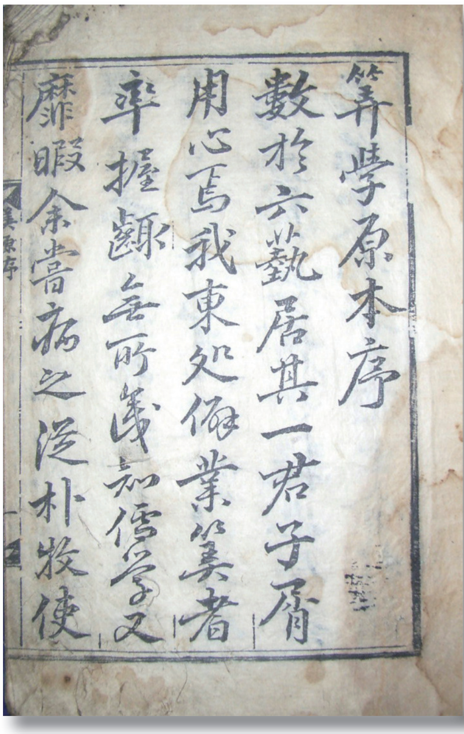

Figure 4. The first page of the preface to *Sanhak Wonbon* (1700) written by Park Yul (1621–1668). This book is the first extant mathematics book with the publishing year in Joseon and is the first one dealing with Tianyuanshu in East Asia after Zhu Shijie's Siyuan Yujian (1303). The preface is written by Choe SeokJeong in his own calligraphy.

Hong JeongHa. When solving a polynomial equation $p(x) = 0$, Shisuo Kaifangfa is based on the binomial expansions of $(y+\alpha)^n$ and Zengcheng Kaifangfa on the successive synthetic divisions of $p(x)$ by $x-\alpha$. Using this connection Hong JeongHa, the great if not the greatest mathematician in Joseon, obtained the binomial coefficients of $a(x+\alpha)^n$ for any a and α using the synthetic divisions of ax^n by $x-\alpha$.

He completed the theory of equations and applied it widely using Tianyuanshu. His book *Guiljib* includes equations of degrees up to and including 10. The most important contribution of Tianyuanshu in Joseon is in the theory of right triangles (Gougushu). Hong's *Guiljib*, together with *Gugo Sulyo* by his contemporary Yu Suseok made a great contribution to the theory before 1713 [1].

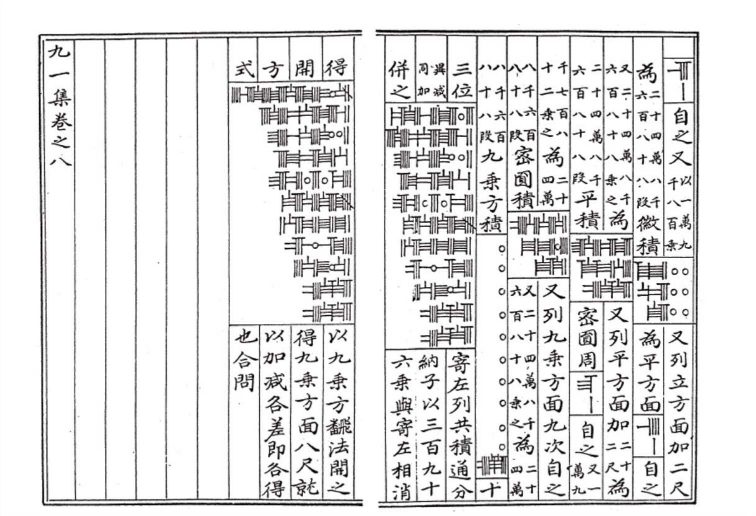

Figure 5. The Tianyuanshu of Hong JeongHa from *Guiljib* (1724). The Tianyuan diagrams show how to obtain an equation of degree 10.

Fully understanding the structure of Zengcheng Kaifangfa, Hong could complete the method exactly like that of the present day — not relying on the positioning of digits of solutions as in the Chinese method. In the process of Zengcheng Kaifangfa, there occur changes of signs of coefficients, called Fanji, Fanfa, or Huangu, and the growth of coefficients in their absolute values with the same signs, called Yiji or Toutai. He also discussed Fanji and Yiji. Hong's method of solving equations in the setting of the field of rational numbers must be praised as perfect.

Li Ye's *Ceyuan Haijing* (1248), which contains the representation of rational polynomials $\sum_{k=m}^{n} a_k x^k$ $(m,n \geq 0)$ using Tianyuanshu, was introduced to Joseon only in the mid-19th century. However, there were many occasions where this method was used in 18th-century Joseon.

Lee SangHyeok also published *Jeongburon*, which is the first part of his *Iksan* and deals with the theory of equations. Lee tried to unify the theory of constructing and solving equations including the theory of systems of polynomial equations of higher degrees. Following Hong JeongHa, Lee gave sufficient conditions for quadratic equations to have Fanji and Yiji.

Joseon mathematicians, with the scanty information in *Yanghui Suanfa* and *Suanxue Qimeng*, were able to build a perfect theory of equations in the setting of the field of rational numbers [1].

4.5 Theory of Approximations

The methods of approximate solutions were initiated by Liu Hui in his commentary of *Jiuzhang Suanshu* and became a very important subject. For the study of approximations one should evaluate the size of errors, which, however, had not gotten any attention in Asia.

Park Yul obtained approximate solutions of quadratic equations $ax^2 - b = 0$ by linear interpolations and then calculated the errors of the approximate solutions for intentionally setting problems with exact rational solutions. The value 1.4 was well accepted and frequently used as an approximation of $\sqrt{2}$. He states in his book that $\sqrt{2}x - 1.4x \leq 1$ holds for $x \leq 70$ when x is the side of a square, and that $\frac{5x}{7} - \frac{x}{\sqrt{2}} > 0.5$ holds for $x > 70$ when x is the diagonal of a square.

Jo TaeGu was also interested in approximations. He was interested in approximations of $\sqrt{2}$, $\sqrt{3}$ where the latter is used for the height of a regular triangle. For better approximations, Jo had $\sqrt{2} \approx \frac{99}{70}, \frac{239}{169}$ and $\sqrt{3} \approx \frac{168}{97}, \frac{97}{56}$. Jo also introduced the earliest concept of limits in Joseon as he explained the method of circle division: "As the division process tends to infinity, the chord, the side of polygon approaches the arc."

Finally, Lee SangHyeok took the series representations for *sine* and *versine* in a *circle* in *Sansul Gwangyun* and said that the number of terms for a partial sum for the alternating series can be determined by the absolute value of the first neglected term.

5 Relations with Neighboring Countries

The three Asian countries China, Korea, and Japan had many cultural exchanges through their long history. The educational system in Unified Silla was adopted in Japan in the early 8th century, and the most important books, *Yanghui Suanfa* and *Suanxue Qimeng*, were passed on to Japan from Joseon in the late 16th century and ignited the Wasan by Seki Takakazu. The Tianyuanshu in Joseon presumably set off the restoration of Song–Yuan mathematics at the end of the 18th century in China. Luo ShiLin of Qing found a copy of the Joseon republication of *Suanxue Qimeng* in a Beijing bookstore and then he could complete the study of Zhu Shijie's mathematics. This implies that Joseon also exported mathematics books to China.

References

[1] Hong Sung Sa, Theory of Equations in the History of Chosun Mathematics, *Proceeding Book 2*, The HPM Satellite Meeting of ICME-12 (2012), 719–731.

[2] Hong Sung Sa, A Brief History of Chosun Mathematics, in RIMS Symposium on Study of the History of Mathematics (Aug. 27–30, 2013), to appear.

[3] Kim Young Wook, A Summary of the History of Mathematics in the 17th Century Korea, in RIMS Symposium on Study of the History of Mathematics (Aug. 27–30, 2013), to appear.

[4] Kim Yong Woon ed., Hanguk Gwahak Gisulsa Jaryo Daegye, Math. Part (Collection of the Historical Sources in the Science and Technology of Korea), Ryeogang Pub. Co. (1985).

[5] Li Yan and Du Shiran, *Chinese Mathematics—A Concise History*, tr. by J. N. Crossley and A. W.-C. Lun, Clarendon Press (1987).