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AN OPERATOR SPLITTING METHOD FOR PRICING THE ELS OPTION

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ABSTRACT. This paper presents the numerical valuation of the two-asset step-down equity-linked securities (ELS) option by using the operator-splitting method (OSM). The ELS is one of the most popular financial options. The value of ELS option can be modeled by a modified Black-Scholes partial differential equation. However, regardless of whether there is a closed-form solution, it is difficult and not efficient to evaluate the solution because such a solution would be represented by multiple integrations. Thus, a fast and accurate numerical algorithm is needed to value the price of the ELS option. This paper uses a finite difference method to discretize the governing equation and applies the OSM to solve the resulting discrete equations. The OSM is very robust and accurate in evaluating finite difference discretizations. We provide a detailed numerical algorithm and computational results showing the performance of the method for two underlying asset option pricing problems such as cash-or-nothing and step-down ELS. Final option value of two-asset step-down ELS is obtained by a weighted average value using probability which is estimated by performing a MC simulation.

1. INTRODUCTION

Equity-linked securities (ELS) are securities whose return on investment is dependent on the performance of the underlying equities linked to the securities. Since ELS were introduced to Korea in 2003, the booming world economy and expanding financial markets have shifted funds previously focused on real estate to new investment vehicles. The ELS option represents one of the new investment vehicles in that they can be used to structure various products according to the needs of investors. We can model the value of the ELS option by a modified Black-Scholes partial differential equation (BSPDE) [1, 3, 10, 11, 12, 13]. Typically, there is no closed-form solution, and even if there were such a solution, evaluating it would be difficult because it would be represented by multiple integrations. Therefore, a fast and accurate numerical algorithm is needed to price the ELS option. We use a finite difference method to discretize the BSPDE and apply the operator-splitting method (OSM) [3, 5] to solve the resulting discrete equations. The basic idea behind the OSM is to reduce multi-dimensional equations into multiple one-dimensional problems. The OSM is very robust and accurate in evaluating finite

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difference discretizations. The rest of the paper is organized as follows. Section 2 provides a basic information and discuss the payoff of two-asset step-down ELS. Section 3 introduces the Black-Scholes model with two underlying assets. Section 4 presents the finite difference discretizations for the BSPDE and a numerical solution algorithm using the OSM. Section 5 presents the computational results showing the performance of the method for option pricing problems with two underlying assets: cash-or-nothing and step-down ELS. Conclusions are presented in Section 6.

2. TWO-ASSET STEP-DOWN ELS

The payoff of two-asset step-down ELS is as follows:

- Early obligatory redemption occurs and a given rate of return is paid if the value of the worst performer is greater than or equal to the prescribed exercise price on the given observation date. Here, Here the worst performer is defined as one of the two underlying assets whose value is lower than that of the other.
- If early obligatory redemptions did not occur until the maturity time, then the return is determined by the Knock-In criterion.

The basic parameters of two-asset step-down ELS are as follows:

- Maturity : T
- Face value : F
- Underlying assets at time t : $x(t)$ and $y(t)$
- Worst performer : $S_t = \min[x(t),\ y(t)]$
- Conditions for early redemption : Let N be the number of observation dates.

Observation date	t_1	t_2	$\cdots$	$t_N = T$
Exercise price	K_1	K_2	$\cdots$	K_N
Rate of return	c_1	c_2	$\cdots$	c_N

Case 1) Early obligatory redemptions happened

If the value of the worst performer S_{t_i} is greater than or equal to the exercise price K_i at time $t = t_i$, then $(1 + c_i)F$ is paid, and the contract expires.

Case 2) Early obligatory redemptions did not happen

Let D denote the Knock-In barrier level and d denote a dummy.

(i) If a Knock-in event does not occur, that is, $m_T = \min\{S_t |\ 0 \le t \le T\} > D$, then $(1 + d)F$ is paid.

(ii) If a Knock-in event occurs, $(1 + S_T/S_0)\, F$ is paid.

We now summarize the payoff function. Let $\chi_i = \chi_{A_i}$, where χ_i denotes the characteristic function of $A_i = \{x \ge K_i \text{ and } y \ge K_i\}$. Here K_i is the exercise price at time t_i. Let $u(x, y, t)$ denote the value of the option. Generally, the payoff function of two-asset step-down ELS is

constructed as follows:

$$u(x,y,t_i) = \begin{cases} \chi_1 = 1 \quad \text{Payoff} = (1+c_1)F \\ \chi_1 = 0 \quad \begin{cases} \chi_2 = 1 \quad \text{Payoff} = (1+c_2)F \\ \chi_2 = 0 \quad \begin{cases} \chi_3 = 1 \quad \text{Payoff} = (1+c_3)F \\ \chi_3 = 0 \quad \begin{cases} \chi_4 = 1 \quad \text{Payoff} = (1+c_4)F \\ \chi_4 = 0 \quad \begin{cases} m_T > D, \text{ then} \\ \text{Payoff} = (1+d)F \\ m_T \le D, \text{ then} \\ \text{Payoff} = (1+S_T/S_0)F \end{cases} \end{cases} \end{cases} \end{cases} \end{cases}$$

In this paper, we chose the following parameters: the reference price $K_0 = 100$, the interest rate $r = 5\%$, the volatilities of the underlying assets $\sigma_1 = 25\%, \sigma_2 = 30\%$, the total time $T = 1$ year, the face price $F = 100$, the Knock-In barrier level $D = 0.6K_0$, and the dummy rate $d = 16\%$. The other parameters are listed in Table 1. Figure 1 shows the profit-and-loss diagram of two-asset step-down ELS.

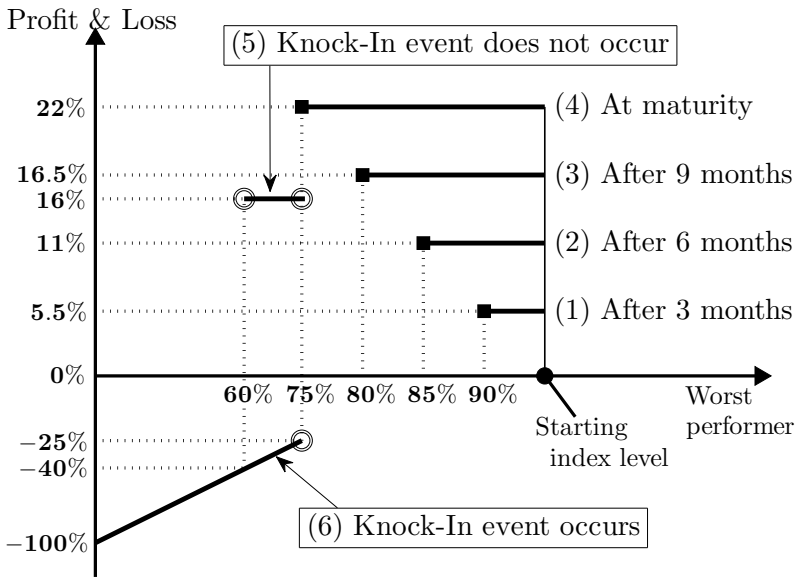


FIGURE 1. Profit-and-loss diagram at early redemption and maturity for two-asset step-down ELS.

Observation date	t_1	t_2	t_3	$t_4 = T$
Exercise price	$K_1 = 0.90K_0$	$K_2 = 0.85K_0$	$K_3 = 0.80K_0$	$K_4 = 0.75K_0$
Return rate	$c_1 = 5.5\%$	$c_2 = 11\%$	$c_3 = 16.5\%$	$c_4 = 22\%$

TABLE 1. Parameters of two-asset step-down ELS.

3. THE BLACK-SCHOLES MODEL WITH TWO UNDERLYING ASSETS

In the Black-Scholes model [2], the underlying assets x and y satisfy the following stochastic differential equations:

$$\begin{aligned} dx(t) &= \mu_1 x(t)dt + \sigma_1 x(t)dW_1, \\ dy(t) &= \mu_2 y(t)dt + \sigma_2 y(t)dW_2, \end{aligned}$$

where μ_1 and μ_2 are the instantaneous expected rates of return, σ_1 and σ_2 are the constant volatilities, and $W_1(t)$ and $W_2(t)$ are the standard Brownian motions of assets x and y, respectively. The terms dW_1 and dW_2 contain randomness which is a key feature of asset prices and assumed to be a Wiener process. The Wiener processes are correlated by $\langle\, dW_1 dW_2 \,\rangle = \rho dt$, where ρ is the correlation value between the two Wiener processes. An increase in ρ results in asymmetry in the distribution of $W_1(t)$ and $W_2(t)$. Then by the Itô lemma and the non-arbitrage principle, the two-dimensional Black-Scholes partial differential equation is

$$\begin{aligned} \frac{\partial u(x,y,t)}{\partial t} &= -\frac{1}{2}\sigma_1^2 x^2 \frac{\partial^2 u(x,y,t)}{\partial x^2} - \frac{1}{2}\sigma_2^2 y^2 \frac{\partial^2 u(x,y,t)}{\partial y^2} - \rho\sigma_1\sigma_2 xy \frac{\partial^2 u(x,y,t)}{\partial x \partial y} \\ &\quad -rx\frac{\partial u(x,y,t)}{\partial x} - ry\frac{\partial u(x,y,t)}{\partial y} + ru(x,y,t), \qquad (3.1) \\ u(x,y,T) &= \Phi(x,y), \end{aligned}$$

where $r > 0$ is a constant as risk-free interest rate and $\Phi(x,y)$ is a payoff function.

4. NUMERICAL SOLUTION

In this section, we describe the numerical discretization of Eq. (3.1). We also present the operator-splitting algorithm in detail.

4.1. **Discretization.**

Let $\mathcal{L}_{BS}$ be the operator

$$\mathcal{L}_{BS} = \frac{1}{2}\sigma_1^2 x^2 \frac{\partial^2 u}{\partial x^2} + \frac{1}{2}\sigma_2^2 y^2 \frac{\partial^2 u}{\partial y^2} + \rho\sigma_1\sigma_2 xy \frac{\partial^2 u}{\partial x \partial y} + rx\frac{\partial u}{\partial x} + ry\frac{\partial u}{\partial y} - ru.$$

Then the two-dimensional Black-Scholes equation can be rewritten as

$$\frac{\partial u}{\partial \tau} = \mathcal{L}_{BS}, \text{ for } (x,y,\tau) \in \Omega \times (0,T],$$

where $\tau = T - t$ and T is the expiration time. The original option pricing problems are defined in the unbounded domain $\{(x,y,\tau) \mid x \geq 0, y \geq 0, \tau \in [0,T]\}$. We truncate this domain into a finite computational domain $\{(x,y,\tau) \mid 0 \leq x \leq L, 0 \leq y \leq M, \tau \in [0,T]\}$, where L and

M are large enough so that the error of the price u arisen by the truncation is negligible. For example, L and M can be two or three times greater than the exercise price [6]. We have the linear boundary conditions [8, 9, 11, 14] for the artificial boundaries

$$\frac{\partial^2}{\partial x^2}u(0,y,\tau)=\frac{\partial^2}{\partial x^2}u(L,y,\tau)=\frac{\partial^2}{\partial y^2}u(x,0,\tau)=\frac{\partial^2}{\partial y^2}u(x,M,\tau)=0,$$
$$\text{for } 0\le x\le L,\ 0\le y\le M \text{ and } 0\le\tau\le T.$$

The numbers of grid steps are denoted by N_x, N_y, and N_τ in the x-, y- and τ-directions, respectively. We first discretize the given computational domain $\Omega=(0,L)\times(0,M)$ as a uniform grid with a space step $h=L/N_x=M/N_y$ and a time step $\Delta\tau=T/N_\tau$. Denote the numerical approximation of the solution by

$$u_{ij}^n\equiv u(x_i,y_j,\tau^n)=u\left((i-0.5)h,(j-0.5)h,n\Delta\tau\right),$$

where $i=1,\dots,N_x$, $j=1,\dots,N_y$ and $n=0,\dots,N_\tau$. We use a cell-centered discretization because we use the following linear boundary condition:

$$u_{0j}=2u_{1j}-u_{2j}, u_{N_x+1,j}=2u_{N_x,j}-u_{N_x-1,j} \text{ for } j=1,\cdots,N_y,$$
$$u_{i0}=2u_{i1}-u_{i2}, u_{i,N_y+1}=2u_{i,N_y}-u_{i,N_y-1} \text{ for } i=1,\cdots,N_x.$$

4.2. **Operator-splitting method.** The basic idea behind the operator-splitting method is to reduce multi-dimensional equations into multiple one-dimensional problems [3, 5]. We introduce the basic OS scheme for the two-dimensional Black-Scholes equation as follows:

$$\frac{u_{ij}^{n+1}-u_{ij}^n}{\Delta\tau} = \mathcal{L}_{BS}^x u_{ij}^{n+\frac12}+\mathcal{L}_{BS}^y u_{ij}^{n+1}, \tag{4.1}$$

where the discrete difference operators $\mathcal{L}_{BS}^x$ and $\mathcal{L}_{BS}^y$ are defined by

$$\begin{aligned}
\mathcal{L}_{BS}^x u_{ij}^{n+\frac12} &= \frac{(\sigma_1 x_i)^2}{2}\frac{u_{i-1,j}^{n+\frac12}-2u_{ij}^{n+\frac12}+u_{i+1,j}^{n+\frac12}}{h^2}\\
&\quad+\lambda_1\sigma_1\sigma_2\rho x_i y_j\frac{u_{i+1,j+1}^n+u_{ij}^n-u_{i,j+1}^n-u_{i+1,j}^n}{h^2}\\
&\quad+rx_i\frac{u_{i+1,j}^{n+\frac12}-u_{ij}^{n+\frac12}}{h}-\lambda_2 r u_{ij}^{n+\frac12},\\
\mathcal{L}_{BS}^y u_{ij}^{n+1} &= \frac{(\sigma_2 y_j)^2}{2}\frac{u_{i,j-1}^{n+1}-2u_{ij}^{n+1}+u_{i,j+1}^{n+1}}{h^2}\\
&\quad+(1-\lambda_1)\sigma_1\sigma_2\rho x_i y_j\frac{u_{i+1,j+1}^{n+\frac12}+u_{ij}^{n+\frac12}-u_{i,j+1}^{n+\frac12}-u_{i+1,j}^{n+\frac12}}{h^2}\\
&\quad+ry_j\frac{u_{i,j+1}^{n+1}-u_{ij}^{n+1}}{h}-(1-\lambda_2)ru_{ij}^{n+1}.
\end{aligned}$$

The first step is implicit in the x-direction, whereas the second step is implicit in the y-direction. The OS scheme moves from the time level n to an intermediate time level $n+\frac{1}{2}$ and then to the time level $n+1$. Through this process, the OS method is to split two problems. We then approximate each subproblem by an implicit scheme:

$$\frac{u_{ij}^{n+\frac{1}{2}} - u_{ij}^{n}}{\Delta\tau} = \mathcal{L}_{BS}^{x} u_{ij}^{n+\frac{1}{2}}, \tag{4.2}$$

$$\frac{u_{ij}^{n+1} - u_{ij}^{n+\frac{1}{2}}}{\Delta\tau} = \mathcal{L}_{BS}^{y} u_{ij}^{n+1}. \tag{4.3}$$

Note that combining two Eqs. (4.2) and (4.3) results in Eq. (4.1). The following describes an algorithm of the OS method.

Algorithm OS

- *Step 1*
  Eq. (4.2) is rewritten as follows. For each j, we have

$$\alpha_i u_{i-1j}^{n+\frac{1}{2}} + \beta_i u_{ij}^{n+\frac{1}{2}} + \gamma_i u_{i+1j}^{n+\frac{1}{2}} = f_{ij}, \tag{4.4}$$

  where

$$\alpha_i = -\frac{1}{2}\frac{\sigma_1^2 x_i^2}{h^2},\ \beta_i = \frac{1}{\Delta\tau} + \frac{\sigma_1^2 x_i^2}{h^2} + \frac{r x_i}{h} + \lambda_2 r,$$
$$\gamma_i = -\frac{1}{2}\frac{\sigma_1^2 x_i^2}{h^2} - \frac{r x_i}{h},\ \text{for } i = 1, ..., N_x$$

  and

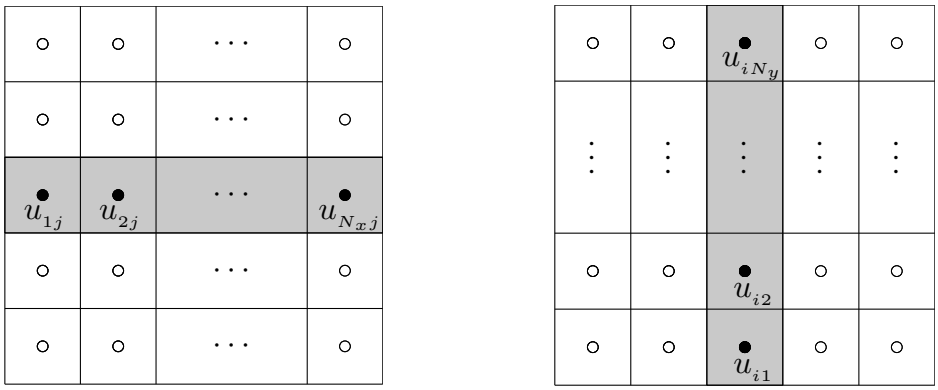
$$f_{ij} = \lambda_1 \rho \sigma_1 \sigma_2 x_i y_j \frac{u_{i+1,j+1}^{n} - u_{i+1,j}^{n} - u_{i,j+1}^{n} + u_{ij}^{n}}{h^2} + \frac{u_{ij}^{n}}{\Delta\tau}. \tag{4.5}$$

  The first step of the OS method is then implemented in a loop over the y-direction:

```
for   j = 1 : N_y
  for   i = 1 : N_x
        Set f_ij by Eq. (4.5)
  end
  Solve A_x u^{n+1/2}_{1:N_x,j} = f_{1:N_x,j} by using Thomas algorithm (see Fig. 2(a))
end
```

Here the matrix A_x is a tridiagonal matrix constructed from Eq. (4.4) with a linear boundary condition

$$A_x = \begin{pmatrix} 2\alpha_1+\beta_1 & \gamma_1-\alpha_1 & 0 & \dots & 0 & 0 \\ \alpha_2 & \beta_2 & \gamma_2 & \dots & 0 & 0 \\ 0 & \alpha_3 & \beta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \beta_{N_x-1} & \gamma_{N_x-1} \\ 0 & 0 & 0 & \dots & \alpha_{N_x}-\gamma_{N_x} & \beta_{N_x}+2\gamma_{N_x} \end{pmatrix}.$$



(a) Step 1 (b) Step 2

FIGURE 2. Two steps of the OSM.

- *Step 2*

As in *Step 1*, Eq. (4.3) is rewritten as follows:

$$\alpha_j u_{i,j-1}^{n+1} + \beta_j u_{ij}^{n+1} + \gamma_j u_{i,j+1}^{n+1} = g_{ij}, \tag{4.6}$$

where

$$\alpha_j = -\frac{1}{2}\frac{\sigma_2^2 y_j^2}{h^2},\ \beta_j = \frac{1}{\Delta\tau} + \frac{\sigma_2^2 y_j^2}{h^2} + \frac{r y_j}{h} + (1-\lambda_2)r,$$
$$\gamma_j = -\frac{1}{2}\frac{\sigma_2^2 y_j^2}{h^2} - \frac{r y_j}{h},\ \text{for } j = 1, ..., N_y$$

and

$$g_{ij} = (1-\lambda_1)\rho\sigma_1\sigma_2 x_i y_j \frac{u_{i+1,j+1}^{n+\frac{1}{2}} - u_{i+1,j}^{n+\frac{1}{2}} - u_{i,j+1}^{n+\frac{1}{2}} + u_{ij}^{n+\frac{1}{2}}}{h^2} + \frac{u_{ij}^{n+\frac{1}{2}}}{\Delta\tau}. \tag{4.7}$$

As with *Step 1*, *Step 2* is then implemented in a loop over the x-direction:

for $i = 1 : N_x$
  for $j = 1 : N_y$
    Set g_{ij} by Eq. (4.7)
  end
  Solve $A_y u^{n+1}_{i,1:N_y} = g_{i,1:N_y}$ by using Thomas algorithm (see Fig. 2(b))
end

Here A_y is tridiagonal matrix constructed from Eq. (4.6) with a linear boundary condition

$$A_y = \begin{pmatrix} 2\alpha_1+\beta_1 & -\alpha_1+\gamma_1 & 0 & \dots & 0 & 0 \\ \alpha_2 & \beta_2 & \gamma_2 & \dots & 0 & 0 \\ 0 & \alpha_3 & \beta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \beta_{N_y-1} & \gamma_{N_y-1} \\ 0 & 0 & 0 & \dots & \alpha_{N_y}-\gamma_{N_y} & \beta_{N_y}+2\gamma_{N_y} \end{pmatrix}.$$

5. COMPUTATIONAL RESULTS

This section presents the convergence test (which determined the accuracy of the OS method) and the numerical experiments for two-asset step-down ELS.

5.1. **Convergence test.** Since the two-asset cash-or-nothing option can be useful building block for constructing more complex and exotic option products, consider the European two-asset cash-or-nothing call option [4]. Given two stock prices x and y, the payoff of the call option is

$$u(x, y, 0) = \begin{cases} \text{Cash} & \text{if} \quad x \geq K_1 \text{ and } y \geq K_2, \\ 0 & \text{otherwise,} \end{cases} \tag{5.1}$$

where K_1 and K_2 are the strike prices of x and y, respectively. The formula for the exact value of the cash-or-nothing option is known [4]. To estimate the convergence rate, we performed numerical simulations with a set of increasingly finer grids up to $T = 1$. We considered a computational domain, $\Omega = [0, 300] \times [0, 300]$. The initial condition was Eq. (5.1) with the strike prices $K_1 = K_2 = 100$ and Cash $= 1$. The volatilities were $\sigma_1 = 0.25$, $\sigma_2 = 0.3$, the correlation was $\rho = 0.5$, and the risk-free interest rate was $r = 0.05$. Also, the weighting factors were $\lambda_1 = \lambda_2 = 0.5$. The error of the numerical solution was defined as $e_{ij} = u^e_{ij} - u_{ij}$ for $i = 1, \cdots, N_x$ and $j = 1, \cdots, N_y$, where u^e_{ij} is the exact solution and u_{ij} is the numerical solution. We computed discrete l^2 norm of the error, $\|\mathbf{e}\|_2$. We also used the root mean square error (RMSE). The RMSE was defined as

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i,j}^{N}\left(u^e_{ij} - u_{ij}\right)^2},$$

where N is the number of points on the gray region in Fig. 3.

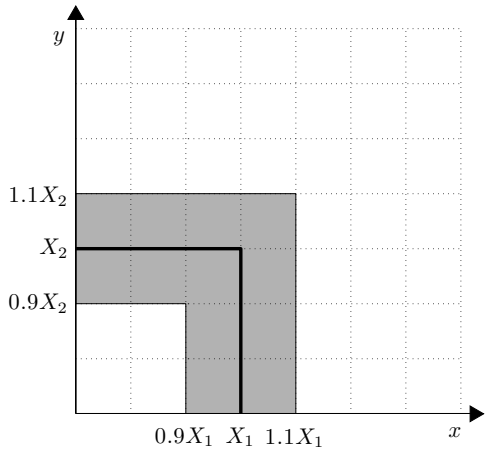


FIGURE 3. The gray region is part where the RMSE is estimated.

Table 2 shows the discrete l^2 norms of the errors in a quarter of the domain, $[0, 150] \times [0, 150]$, the RMSE which is estimated in the gray region shown in Fig. 3 and the rates of convergence for $\|\mathbf{e}\|_2$ and RMSE. The results suggest that the scheme has first-order accuracy and the RMSE has second-order accuracy in space and time.

Mesh	h	Δt	$\|\mathbf{e}\|_2$	order	RMSE	order
128×128	2.3437	0.1000	0.005344		0.000177	
256×256	1.1719	0.0500	0.002716	0.9764	0.000053	1.7397
512×512	0.5859	0.0250	0.001335	1.0246	0.000011	2.2685
1024×1024	0.2930	0.0125	0.000679	0.9754	0.000003	1.8745

TABLE 2. Convergence test.

5.2. **Numerical test of a two-asset step-down ELS.** Let u and v be the solutions with payoffs which knock-in event happens and does not happen, respectively. Fig. 4(a) and (b) show the initial configurations of u and v, respectively.

And Fig. 5(a) and (b) show the final profiles of u and v, respectively, at $T = 1$ with $N_x = N_y = 100$, $K_0 = 100$, $L = 300$, and the parameters listed in Table. 1.

The final two-asset step-down ELS price is obtained by a weighted average of u and v by each probability. By performing a Monte Carlo (MC) simulation [7] for 20000 samples, we estimated that a knock-in event occurs with a probability of approximately 0.1. Therefore, we defined the final ELS value as $0.1u + 0.9v$. Fig. 6 (a) shows the weighted average value $0.1u + 0.9v$, and (b) shows the overlapped contour lines of the weighted average values.

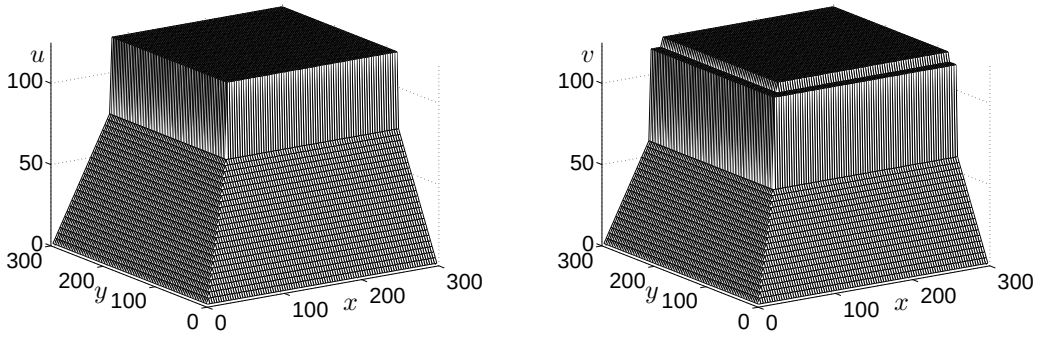


FIGURE 4. Initial conditions for u and v, respectively.

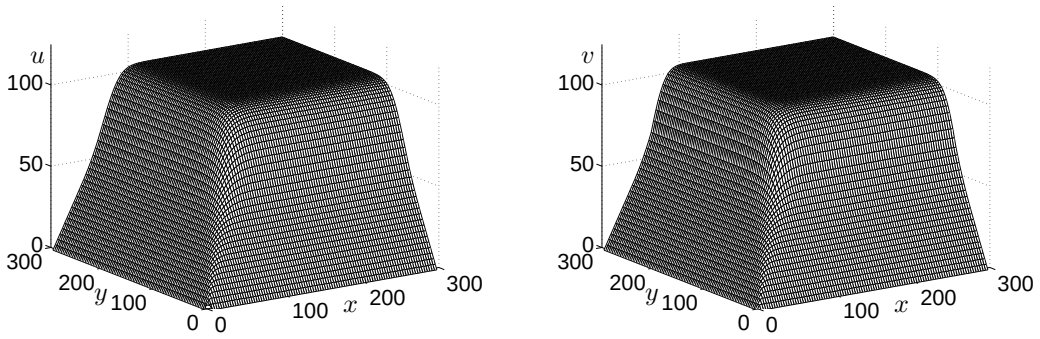
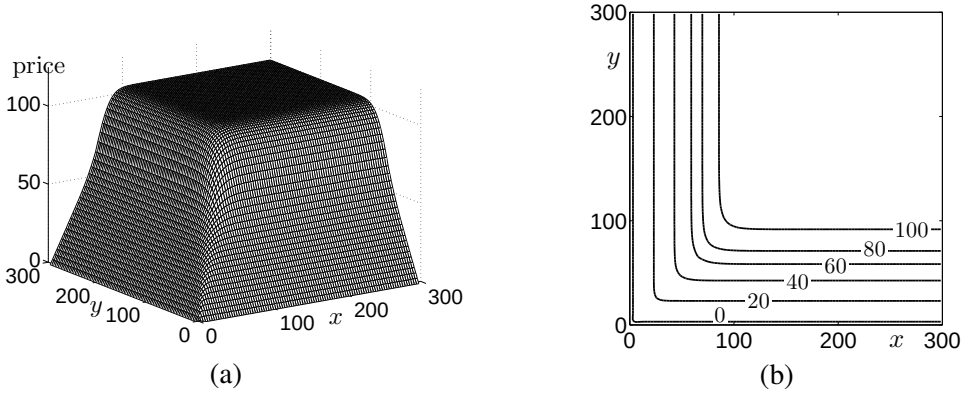


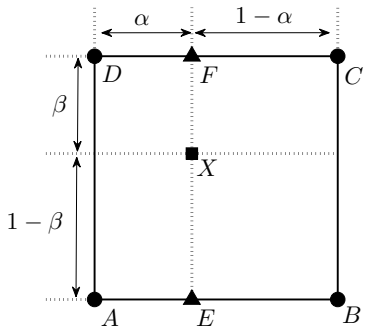
FIGURE 5. Numerical results for u and v, respectively, at $T = 1$.



(a) (b)

FIGURE 6. (a) The weighted average value $0.1u + 0.9v$ at $T = 1$. (b) The contour lines of the weighted average values.

Usually, the position of current underlying assets does not coincide with the numerical grid points. Therefore, we needed to use an interpolation method. As shown in Fig. 7, we obtained the numerical values at the specific point X by using the bilinear interpolation.



$$
\begin{aligned}
E &= (1-\alpha)A + \alpha B \\
F &= (1-\alpha)D + \alpha C \\
\therefore X &= (1-\beta)F + \beta E
\end{aligned}
$$

FIGURE 7. A diagram of the bilinear interpolation: the specific value X is obtained from the numerical solutions A, B, C, and D near the specific point X by the bilinear interpolation.

Table 3 shows the results for two-asset step-down ELS obtained using the OSM at the point $(100, 100)$ with different meshes and time steps.

Mesh	N_t	$v(100,100)$	$u(100,100)$	Weighted average $0.1u + 0.9v$
300×300	365	103.041093	101.306561	102.867640
600×600	730	103.028876	101.359551	102.861944
1200×1200	1460	103.007394	101.369623	102.843617
2400×2400	2920	102.987068	101.361671	102.824528

TABLE 3. Two-asset step-down ELS prices u, v, and the weighted average value $0.1u+0.9v$ obtained using the OSM at the point $(100, 100)$ with different meshes and time steps.

Fig. 8 shows the two-asset step-down ELS price at position $(x, y) = (100, 100)$ obtained using the OSM and the MC simulation. The solid line is the result obtained using the OSM with a 2400×2400 mesh. The symbol lines are the results from three trial MC simulations with an increasing number of samples. Generally, MC simulations in computational finance are easy to apply than the FDM. Because results obtained using the MC simulation are affected by the distribution of random numbers, the accuracy of MC simulation can be guaranteed through many trials.

6. CONCLUSIONS

In this paper, we presented a numerical algorithm for the two-asset step-down ELS option by using the OSM. We modeled the value of ELS option by using a modified Black-Scholes

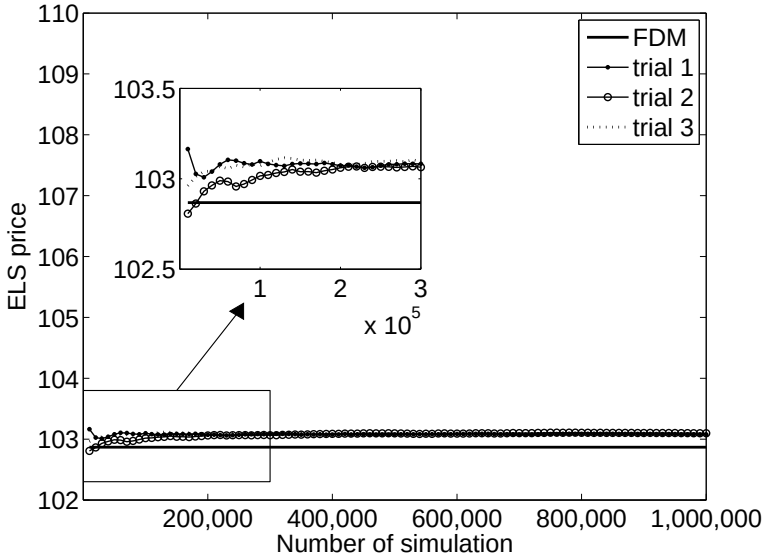


FIGURE 8. Two-asset step-down ELS price obtained using the OSM and the Monte-Carlo simulation versus the number of simulations.

partial differential equation. A finite difference method was used to discretize the governing equation, and the OSM was applied to solve the resulting discrete equations. We provided a detailed numerical algorithm and computational results demonstrating the performance of the method for two underlying asset option pricing problems such as cash-or-nothing and step-down ELS. In addition, we applied a weighted average value with a probability obtained using the MC simulation to obtain the option value of two-asset step-down ELS.

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