기하학 개론

Ch.2. 구면 위의 기하.

문제 1.1. $z = \sqrt{1-x^2-y^2}$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1+z_x^2+z_y^2}\, dy\, dx$$

$$z_x = \frac{d}{dx} z = \frac{-2x}{2\sqrt{1-x^2-y^2}} = \frac{-x}{\sqrt{1-x^2-y^2}}$$

$$z_y = \frac{d}{dy} z = \frac{-y}{\sqrt{1-x^2-y^2}}$$

$$\sqrt{1+z_x^2+z_y^2} = \sqrt{1+\left(\frac{-x}{\sqrt{1-x^2-y^2}}\right)^2+\left(\frac{-y}{\sqrt{1-x^2-y^2}}\right)}$$

$$= \sqrt{\frac{(1-x^2-y^2)+x^2+y^2}{1-x^2-y^2}}$$

$$= \frac{1}{\sqrt{1-x^2-y^2}}$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1+z_x^2+z_y^2}\, dy\, dx = \int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{1-x^2-y^2}}\, dy\, dx$$

$$= \int_0^1 \int_0^a \frac{1}{\sqrt{a^2-y^2}}\, dy\, dx \qquad a = \sqrt{1-x^2}$$

$$= \int_0^1 \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{a^2-(a\sin\theta)^2}}\, a\cos\theta \cdot d\theta\, dx \qquad \left(\begin{array}{l} y = a\sin\theta \\ dy = a\cos\theta\, d\theta \end{array}\right.$$

$$= \int_0^1 \int_0^{\frac{\pi}{2}} d\theta\, dx$$

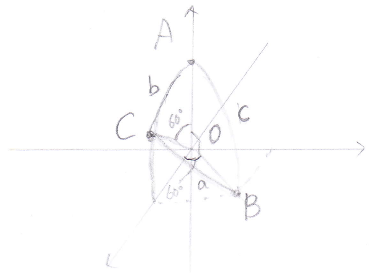
$$= \int_0^1 \frac{\pi}{2}\, dx$$

$$= \frac{\pi}{2}$$

문제 1.2.

문제 2.1

문제 3.1



$A = (1, 0, 0)$

$C = \vec{OC} = (\sin b, 0, \cos b) = (\frac{\sqrt{3}}{2}, 0, \frac{1}{2})$

$B = \vec{OB} = (\sin c \cos A, \sin c \sin A, \cos c) = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$

$$\cos a = \langle \vec{OB}, \vec{OC} \rangle$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + 0 \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot 0$$
$$= \frac{\sqrt{3}}{4}$$

문제 3.2

(1) $X \times (Y \times Z) = \langle X, Z \rangle Y - \langle X, Y \rangle Z$

Let
$X = (x_1, x_2, x_3)$
$Y = (y_1, y_2, y_3)$
$Z = (z_1, z_2, z_3)$

Using
$(a_1, a_2, a_2) \times (b_1, b_2, b_3) = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$
$\langle (a_1, a_2, a_3), (b_1, b_2, b_3) \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$X \times (Y \times Z) = X \times (y_2 z_3 - y_3 z_2, y_3 z_1 - y_1 z_3, y_1 z_2 - y_2 z_1)$$
$$= (x_2(y_1 z_2 - y_2 z_1) - x_3(y_3 z_1 - y_1 z_3), x_3(y_2 z_3 - y_3 z_2) - x_1(y_1 z_2 - y_2 z_1), x_1(y_3 z_1 - y_1 z_3) - x_2(y_2 z_3 - y_3 z_2))$$
$$= (x_2 y_1 z_2 + x_3 y_1 z_3 - x_2 y_2 z_1 - x_3 y_3 z_1, x_1 y_2 z_1 + x_3 y_2 z_3 - x_1 y_1 z_2 - x_3 y_3 z_2, x_1 y_3 z_1 + x_2 y_3 z_2 - x_1 y_1 z_3 - x_2 y_2 z_3) \cdots (*)$$

$\langle X, Z \rangle Y = (x_1 z_1 + x_2 z_2 + x_3 z_3)(y_1, y_2, y_3)$

$-) \langle X, Y \rangle Z = (x_1 y_1 + x_2 y_2 + x_3 y_3)(z_1, z_2, z_3)$

$$\langle X, Z \rangle Y - \langle X, Y \rangle Z = ((x_1 z_1 + x_2 z_2 + x_3 z_3) y_1 - (x_1 y_1 + x_2 y_2 + x_3 y_3) z_1, (x_1 z_1 + x_2 z_2 + x_3 z_3) y_2 - (x_1 y_1 + x_2 y_2 + x_3 y_3) z_2, (x_1 z_1 + x_2 z_2 + x_3 z_3) y_3 - (x_1 y_1 + x_2 y_2 + x_3 y_3) z_3)$$
$$= (x_2 y_1 z_2 + x_3 y_1 z_3 - x_2 y_2 z_1 - x_3 y_3 z_1, x_1 y_2 z_1 + x_3 y_2 z_3 - x_1 y_1 z_2 - x_3 y_3 z_2, x_1 y_3 z_1 + x_2 y_3 z_2 - x_1 y_1 z_3 - x_2 y_2 z_3) \cdots (**)$$

/ Hence $(*) = (**)$ $\square$