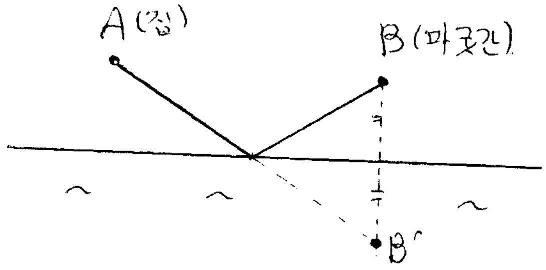
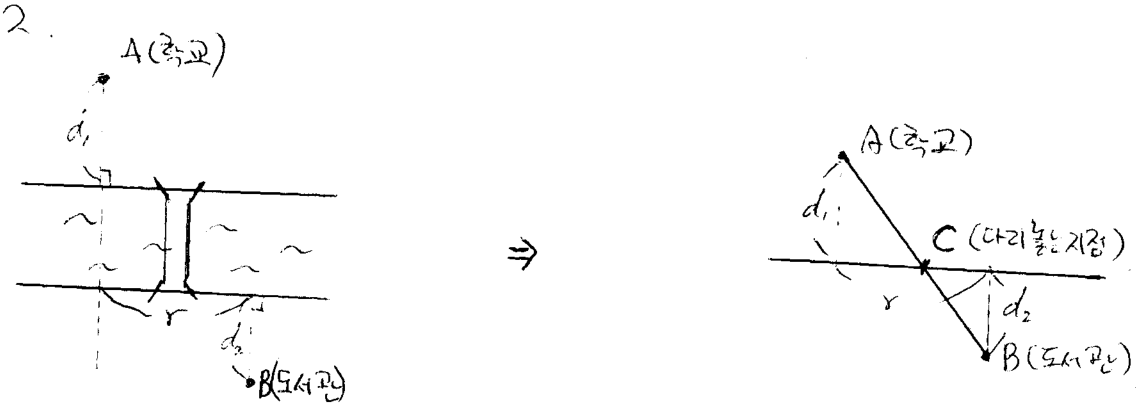
문제 4.1.



문제 4.2.



만약 학교과 도서관이 강가에 있다면 어디에 다리를 놓아도 거리는 같다.
대각선 길을 만들어 갈 수 없는 경우, 지형, 등의 요소에 따라 변화할 수 있음.

문제 4.3.

문제 4.4

문제 4.5.

(1)

α	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	2π
n	7	5	3	2	0 or 1

(2) $\frac{2\pi}{\alpha}-1$

(3)

문제 4.6.

$$R_\theta \circ T_{R_{(-\theta)}(\mathbf{p})}(\mathbf{x}) = R_\theta(\mathbf{x} + R_{(-\theta)}(\mathbf{p}))$$
$$= R_\theta(\mathbf{x}) + R_\theta(R_{(-\theta)}(\mathbf{p}))$$
$$= R_\theta(\mathbf{x}) + \mathbf{p}$$
$$= T_{\mathbf{p}} \circ R_\theta(\mathbf{x})$$

문제 4.7. $D_\ell = T_{2y} \circ R_{2\phi} \circ D_0$

i) Let $\ell : y = ax + b$, $D_\ell(x, y) = (u, v)$

then
$$\begin{cases} \dfrac{y+v}{2} = a \cdot \dfrac{x+u}{2} + b \\[2ex] \dfrac{v-y}{u-x} = -\dfrac{1}{a} \end{cases}$$

→ 중점은 직선의 방정식 만족

→ 수직인 두 직선의 기울기의 곱=(-1)

$$\Rightarrow \begin{cases} u = \dfrac{2a}{a^2+1}y - \dfrac{a^2-1}{a^2+1}x - \dfrac{2ab}{a^2+1} \\[2ex] v = \dfrac{a^2-1}{a^2+1}y + \dfrac{2a}{a^2+1}x + \dfrac{2b}{a^2+1} \end{cases} \quad \cdots \ (*)$$

ii) $(x, y) \xmapsto{D_0} (x, -y)$

$$(x, -y) \xmapsto{R_{2\phi}} \begin{pmatrix} \cos 2\phi & -\sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix} \begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} x\cos 2\phi + y \sin 2\phi \\ x \sin 2\phi - y\cos 2\phi \end{pmatrix}$$

$$(x\cos 2\phi + y\sin 2\phi,\ x\sin 2\phi - y\cos 2\phi)$$

T_{2y}, $y = (r, w)$

$$(x\cos 2\phi + y \sin 2\phi + 2r,\ x\sin 2\phi - y\cos 2\phi + 2w) \text{ — ※}$$

Since $a = \tan\phi$

$$a^2+1 = \tan^2\phi + 1 = \sec^2\phi.$$

$$\frac{2a}{a^2+1} = \frac{2\cdot\tan\phi}{\sec^2\phi} = 2\cdot\sin\phi\cos\phi = \sin 2\phi \quad \cdots ①$$

$$\frac{a^2-1}{a^2+1} = \frac{\tan^2\phi-1}{\sec^2\phi} = \sin^2\phi - \cos^2\phi = -\cos 2\phi. \quad \cdots ②$$

$$\|y\| = d = \frac{|b|}{\sqrt{a^2+1}}$$

$\uparrow$

l과 (0,0) 사이의 거리.

$$y = \left(\frac{b}{\sqrt{a^2+1}}\cos\left(\frac{\pi}{2}+\phi\right), \frac{b}{\sqrt{a^2+1}}\sin\left(\frac{\pi}{2}+\phi\right)\right)$$

$$= \left(\frac{b}{\sqrt{a^2+1}}\cdot(-\sin\phi), \frac{b}{\sqrt{a^2+1}}\cos\phi\right)$$

$$= \left(\frac{b}{\sqrt{a^2+1}}\cdot\frac{-a}{\sqrt{a^2+1}}, \frac{b}{\sqrt{a^2+1}}\cdot\frac{1}{\sqrt{a^2+1}}\right).$$

$$= \left(\frac{-ab}{a^2+1}, \frac{b}{a^2+1}\right) \quad \cdots ③$$

$$= (r, \omega)$$

※ $\leftarrow$ ①, ②, ③

$$\to \left(x\left(-\frac{a^2-1}{a^2+1}\right) + y\cdot\left(\frac{2a}{a^2+1}\right) + 2\cdot\left(\frac{-ab}{a^2+1}\right), x\left(\frac{2a}{a^2+1}\right) + y\cdot\left(\frac{a^2-1}{a^2+1}\right) + 2\cdot\left(\frac{b}{a^2+1}\right)\right)$$

$$= \left(-\frac{a^2-1}{a^2+1}x + \frac{2a}{a^2+1}y + \frac{-2ab}{a^2+1}, \frac{2a}{a^2+1}x + \frac{a^2-1}{a^2+1}y + \frac{2b}{a^2+1}\right) \cdots (**)$$

Hence $(*) = (**)$