1

It is less obvious that division is also possible. We wish to show that $\alpha+i\beta/(\gamma+i\delta)$ is a complex number, provided that $\gamma+i\delta \neq 0$. If the quotient is denoted by $x+iy$, we must have

$$\alpha+i\beta=(\gamma+i\delta)(x+iy).$$

this condition can be written

$$\alpha+i\beta=(\gamma x-\delta y)+i(\delta x+\gamma y),$$

and we obtain the two equations

$$\alpha=\gamma x-\delta y$$
$$\beta=\delta x+\gamma y.$$

This system of simultaneous liear equations has the unique solution

$$x=\frac{\alpha\gamma+\beta\delta}{\gamma^2+\delta^2}$$
$$y=\frac{\beta\gamma-\alpha\delta}{\gamma^2+\delta^2}$$

for we know that $\gamma^2+\delta^2$ is not zero. We have thus the result

$$\frac{\alpha+i\beta}{\gamma+i\delta}=\frac{\alpha\gamma+\beta\delta}{\gamma^2+\delta^2}+i\frac{\beta\gamma-\alpha\delta}{\gamma^2+\delta^2} \quad (1)$$

2

Let $\mathcal{M}$ be a projective plane. Define a new interpretation $\mathcal{M}'$ by taking as "points" of $\mathcal{M}'$ the lines of $\mathcal{M}$ and as "lines" of $\mathcal{M}'$ the points of $\mathcal{M}$, with the same incidence relation. ①Prove that $\mathcal{M}'$ is also a projective plane(called the *dual plane* of $\mathcal{M}$). ②Suppose further that $\mathcal{M}$ has only finitely many points. Prove that all lines in $\mathcal{M}$ have the same number of points lying on them.

3

By combining (1) and (2) we obtain

$$\sum_{\substack{k \ll l; \\ (\beta+m+1)k+(n-1)l \gg \ln \lambda}} ||T_{k,l}||_{L^2 \to L^2} \tag{5}$$

4

Since this summation is uniformly convergent on $\partial \Delta_R(a)$,

$$\frac{1}{2\pi i} \int_\Gamma \frac{f(\zeta)}{\zeta - z} \, d\zeta = \sum_{n=0}^{\infty} \left[\frac{1}{2\pi i} \int_\Gamma \frac{f(\zeta)}{(\zeta - a)^{n+1}} \, d\zeta \right] (z-a)^n.$$

for all z such that $|z - a| < r$.

5

$$M_1 = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & a & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

,

$$\begin{bmatrix} \cdots & C_s & \cdots \end{bmatrix} M_1 = \begin{bmatrix} \cdots & aC_s & \cdots \end{bmatrix}$$