

1

It is less obvious that division is also possible. We wish to show that $\alpha + i\beta/(\gamma + i\delta)$ is a complex number, provided that $\gamma + i\delta \neq 0$. If the quotient is denoted by $x + iy$, we must have

$$\alpha + i\beta = (\gamma + i\delta)(x + iy).$$

this condition can be written

$$\alpha + i\beta = (\gamma x - \delta y) + i(\delta x + \gamma y),$$

and we obtain the two equations

$$\alpha = \gamma x - \delta y$$

$$\beta = \delta x + \gamma y.$$

This system of simultaneous linear equations has the unique solution

$$x = \frac{\alpha\gamma + \beta\delta}{\gamma^2 + \delta^2}$$

for we know that $\gamma^2 + \delta^2$ is not zero. We have thus the result

$$\frac{\alpha + i\beta}{\gamma + i\delta} = \frac{\alpha\gamma + \beta\delta}{\gamma^2 + \delta^2} + i\frac{\beta\gamma - \alpha\delta}{\gamma^2 + \delta^2} \quad (1)$$

2

Let \mathcal{M} be a projective plane. Define a new interpretation \mathcal{M}' by taking as “points” of \mathcal{M}' the lines of \mathcal{M} and as “lines” of \mathcal{M}' the points of \mathcal{M} , with the same incidence relation. ① Prove that \mathcal{M}' is also a projective plane (called the *dual plane* of \mathcal{M}). ② Suppose further that \mathcal{M} has only finitely many points. Prove that all lines in \mathcal{M} have the same number of points lying on them.

3

By combining (1) and (2) we obtain

$$\sum_{\substack{k \leq l, \\ (\beta+m+1)k+(n-1)l \gg \ln \lambda}} \|T_{k,l}\|_{L^2 \rightarrow L^2} \quad (2)$$

4

Since this summation is uniformly convergent on $\partial\Delta_R(a)$,

$$\frac{1}{2\pi i} \int_{\Gamma} \frac{f(\zeta)}{\zeta - z} d\zeta = \sum_{n=0}^{\infty} \left[\frac{1}{2\pi i} \int_{\Gamma} \frac{f(\zeta)}{(\zeta - a)^{n+1}} d\zeta \right] (z - a)^n.$$

for all z such that $|z - a| < r$.

5

$$M_1 = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & a & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

,

$$\left[\cdots \quad C_s \quad \cdots \right] M_1 = \left[\cdots \quad aC_s \quad \cdots \right]$$